Exercise Sheet 9
Advanced Quantum Theory
WS 2010/11

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Exercise 1: **(3 points)**

Show that $\Gamma(a^*) = (\mathrm{J}\Gamma(a))^\perp$ in II.5.2 Nr.7.

Exercise 2: **(3 points)**

Show that $a \subset a^*$ in example II.5.3 Nr.5.

Exercise 3: **(10 points)**

Repetition:
Consider a particle in a potential well in one dimension given by

$$V(x) = -V_0\Theta(a - |x|) = \begin{cases} 0 & x < -a & \text{region I} \\ -V_0 & -a < x < a & \text{region II} \\ 0 & x > a & \text{region III} \end{cases}$$

where $V_0 > 0$.
Find the energy eigenvalues in the range $-V_0 < E < 0$ (bound states) by solving the time independent Schrödinger equation.

(a) Because of reflection symmetry one looks for even and odd solutions and makes the ansatz

$$\underline{\text{even}}: \begin{cases} I: & \psi_1 = e^{kx} \\ II: & \psi_2 = A\cos(qx) \\ III: & \psi_3 = e^{-kx} \end{cases}$$

$$\underline{\text{odd}}: \begin{cases} I: & \psi_1 = e^{kx} \\ II: & \psi_2 = A\sin(qx) \\ III: & \psi_3 = e^{-kx} \end{cases}$$

Check that this ansatz satisfies suitable boundary conditions and gives wave functions that are normalizable. Find relations that connect k and q with E. Show that k and q satisfy $k^2 + q^2 = C^2$ where C is independent of E.

(b) We require that ψ and ψ' (defined piecewise on the three regions) are continuous functions. Use this condition together with $k^2 + q^2 = C^2$ from a) to determine nontrivial solutions for the parameters k, q. Solve the resulting set of equations graphically. Find the number of even (odd) eigenstates for energies $-V_0 < E < 0$.

(c) Plot the eigenfunctions for a potential well with four even and three odd solutions. Characterize these functions according to the number of nodes (zeros) their wave functions have.

(d) In the limit $V_0 \to \infty$ one recovers exercise 3 from sheet 7. Find the form of the even and odd wave functions in the limit $V_0 \to \infty$. Give the corresponding energy eigenvalues, in particular the ground state energy.