

Exercise Sheet 7
Advanced Quantum Theory
WS 2010/11

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Exercise 1: **(2 points)**

Why is the graph of a continuous mapping $\mathbb{E} \rightarrow \mathbb{E}$ from a Banach space \mathbb{E} to itself closed in $\mathbb{E} \times \mathbb{E}$.

Exercise 2: **(4 points)**

Let a be a Hermitian operator.

Show that the existence of $(a \pm i)^{-1}$ implies that of $(a - z)^{-1}$ for all z with $\text{Im}(z) \neq 0$, and that $\|(a - z)^{-1}\| \leq |\text{Im}(z)|^{-1}$.

Exercise 3: **(5 points)**

Repetition:

- (a) A quantum mechanical particle of mass m is confined to a box $0 \leq x \leq a$ in one dimension. This means that its wave function vanishes outside the box for all times.

Find solutions of the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t),$$
$$\psi(x, t) = 0 \quad \text{for all } x < 0 \text{ and } x > a$$

using the product ansatz $\psi(x, t) = f(t)g(x)$.

- (b) Calculate the probability density $\rho(x, t) = |\psi(x, t)|^2$ and the expectation value

$$\langle x \rangle = \frac{\int \psi^*(x, t) x \psi(x, t) dx}{\int \psi^*(x, t) \psi(x, t) dx}.$$