Physics of Soft and Biological Matter I (WS 2013/2014)

Worksheet 7
Depletion interactions and Total Internal Reflection Microscopy

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Important remarks

- You can access the ICP web page to download important materials via the username **icp** and the password **icp**.
- Due date (this time a bit longer): **Wednesday, February 5th, 2014, 12:00**
- You can either send a PDF file to Jens Smiatek (smiatek@icp.uni-stuttgart.de), Stefan Kesselheim (Stefan.Kesselheim@icp.uni-stuttgart.de) or submit a hand-written copy.
- Please write your name on each page and make sure that the pages are easily readable.
- If you have further questions, contact Jens Smiatek (smiatek@icp.uni-stuttgart.de) or Stefan Kesselheim (Stefan.Kesselheim@icp.uni-stuttgart.de).
- The solutions will be discussed on Thursday, February 6th, 2013 at 8:00.

1 Depletion Interactions (7 points)

Consider two big spheres with radius R in a bath of small spheres with radius r. The number density of small spheres is $\rho = N/V$. All spheres are ideal hard spheres, so the system is purely entropic.

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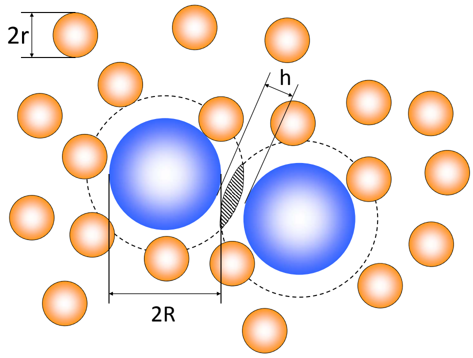


a) (2 points) Calculate the change of free energy ΔF_{SS} as a function of the distance h of the surfaces of the two big spheres. The small spheres can be considered as an ideal gas. When is this approximation valid?

b) (2 points) State the expression in the case $R = r$.

c) (1 point) Compare the result to the free energy ΔF_{SW} for a sphere-wall interaction derived in the lecture.

d) (2 points) In the literature, sometimes the following more handy approximation is used:

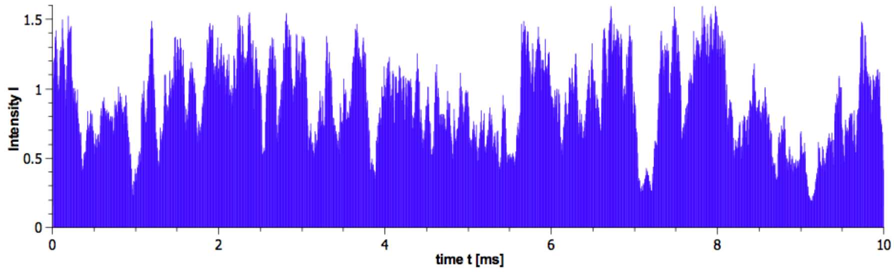
$$\Delta F_{KK} = -3k_B T \Phi_s \frac{R}{\sigma}\left(1 - \frac{h}{\sigma}\right)$$

Here $\sigma = 2r$ is the diameter of the small particles and ϕ_s the volume fraction of small spheres. Compare this expression to your results. Which terms have been omitted?

2 Total Internal Reflection Microscopy (7 points)

You have learned about the concept of Total Internal Reflection Microscopy (TIRM) in the lecture. In TIRM the evanescent waves reflected by a Colloid are used to determine the colloid's position. The relationship between the intensity of the scattered light and the distance used to reconstruct the distance distribution from the intensity distribution. Write a small program in PYTHON or Matlab to perform this analysis with the data available from the ICP homepage. Using `numpy` histograms this programm is literally only a few lines long. **Hint**: When writing the program, do not use the full file, but only the first 100 lines. This makes it faster and easier to debug. You can do this with the command

```
head -n 100 tirm.dat > tirm_cut.dat
```



a) (3 points) Reconstruct the trajectory of the particle, by using the relation

$$I = I_0 \exp(-\beta z)$$

between the intensity I and the position z. Here $\beta = 1/0.147 \mu m$ is the penetration depth. Plot a part of the trajectory. **Hint**: As you don't know I_0, you obtain a free parameter z_0.

b) (2 points) Determine the spatial distribution $P\left(z - z_0\right)$ of the particle.

c) (2 points) Reconstruct the potential by inverting the Boltzmann distribution

$$P(z) = P_0 \exp\left(-\frac{\Phi(z)}{k_B T}\right).$$

Discuss the shape of the potential.

3 Literature (6 points)

S. Sacanna et al., *Lock and key colloids*, Nature **464**, 575-578 (2010).
Read the article and be prepared to present the most important points in the tutorial.