Exercise Sheet 4
Advanced Quantum Theory
WS 2010/11

Fakultät Mathematik und Physik
Universität Stuttgart
Prof. Dr. R. Hilfer

Exercise 1: **(3 points)**

Find an example with 2×2 matrices for which $0 \leq a \leq b$ does not imply $a^2 \leq b^2$. (Hint: $a \geq 0$ if and only if $a = a^*$, $\mathrm{tr}(a) \geq 0$ and $\det(a) \geq 0$.)

Exercise 2: **(3 points)**

Consider two states w_1, w_2. Show that, if $w = \alpha w_1 + (1-\alpha) w_2$ with $0 < \alpha < 1$, then $(\Delta_w a)^2 \geq \alpha (\Delta_{w_1} a)^2 + (1-\alpha)(\Delta_{w_2} a)^2$, and that equality holds if and only if $w_1(a) = w_2(a)$. Here $(\Delta_w a)^2 = w(a^2) - w(a)^2$ is the mean square deviation of the observable a in the state w.

Exercise 3: **(3 points)**

Let $0 \leq a \leq b$. Show that

(i) if a^{-1} exists, then $b^{-1} \leq a^{-1}$,

(ii) if $\ln(a)$ exists, then $\ln(a) \leq \ln(b)$.

(Hint: Use, that $b \geq a \geq 0$ implies

$$\int_0^\infty \sigma(\lambda)(a+\lambda)^{-1} d\lambda \geq \int_0^\infty \sigma(\lambda)(b+\lambda)^{-1} d\lambda$$

for $\sigma \geq 0$.)