Exercise Sheet 1

Advanced Quantum Theory

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Exercise 1: **(4 points)**

A linear mapping $a : \mathbb{E} \to \mathbb{F}$ is bounded if and only if it maps bounded sets to bounded sets, and thus

$$\|a\| := \sup_{\|x\|=1} \|ax\|_{\mathbb{F}} < \infty$$

holds.
Show that for linear mappings the properties

(i) continuous

(ii) continuous at the origin

(iii) bounded

are equivalent.

Exercise 2: **(4 points)**

Let $\mathbb{E}$ and $\mathbb{F}$ be two Banach spaces.
Show that the space of continuous linear mappings $\mathbb{E} \to \mathbb{F}$ (with the uniform topology) is also a Banach space.

Exercise 3: **(4 points)**

Let $a : \mathbb{E} \to \mathbb{F}$ be a continuous linear mapping of two Hilbert spaces.
Show that $a^* : \mathbb{E} \to \mathbb{F}$ is also continuous.

Exercise 4: **(4 points)**

Prove that on a Hilbert space $\mathbb{E}$ the equality

$$\|ax\| = \|a^*x\| = \|x\| \quad \forall x \in \mathbb{E}$$

holds if and only if

$$aa^* = a^*a = 1.$$