Exercise Sheet 8
Advanced Quantum Theory
WS 2010/11

Fakultät Mathematik und Physik
Universität Stuttgart
Prof. Dr. R. Hilfer

Exercise 1: **(3 points)**

Prove that an everywhere defined continuous operator is closed and that a continuous and closed operator is everywhere defined.

Exercise 2: **(4 points)**

Prove Theorem II.4.18 from the lecture.

Exercise 3: **(10 points)**

Repetition:

(a) The time independent Schrödinger equation for the harmonic oscillator in one dimension is given by

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2}{2}x^2\right)\varphi(x) = E\varphi(x).$$

Explain the meaning of the symbols and introduce new coordinates $\xi = \sqrt{\frac{m\omega}{\hbar}}x$.

(b) Make the ansatz $\psi(x) = v(\xi)e^{-\xi^2/2}$ and find the equation for $v(\xi)$.

(c) Assume that $v(\xi)$ behaves as $v(\xi) \sim \xi^n$ for $\xi \to \infty$.

Show that $\psi(\xi) \sim \xi^n e^{-\xi^2/2}$ has the correct asymptotic behaviour and that it gives the correct eigenvalues.

(d) Make a polynomial ansatz for $v(\xi) = \sum_{i=1}^{i_{max}} c_i \xi^i$ and find the recursion relation for c_i. Why is it necessary to introduce a finite order i_{max}?

(e) From the choice of $i_{max} = n$ deduce the eigenvalue e_n and show that the solutions $v_n(\xi)$ are symmetric or antisymmetric polynomials. What are the lowest powers that occur?