1 Introduction

Active Brownian particles perform a motion superposed by Brownian diffusion and active swimming. In the last decades several biological motivated artificial microswimmer were fabricated which exhibit such a motion. Those artificial microswimmers and their biological counterparts show without the presence of external fields at short time scales a directed motion but the motion is randomized by rotational diffusion on longer time scales to a random walk [3]. In nature common bacteria, such as Paramecium, are perfect examples for active particles. But instead of revealing a random walk on long time scales they exhibit a directed motion under the presence of an external field as gravity or external flow. This review focuses on the behaviour of active particles in an external gravitational field (Sec. 2) and an external flow (Sec. 3). Section 2 is based on the papers [2], [4], [6] and section 3 on [5], [6], [7].

Under gravity spherical active particles show equilibrium-like sedimentation profiles [4], but also develop polar order [1]. However, by breaking the symmetry of the particles they are able to move against the gravitational field and invert the sedimentation profile [2].

Similar behaviour occurs in the presence of external flow. Instead of being simply advected by the flow, spherical active swimmers reveal a swinging and tumbling motion stream upwards and downwards [6], whereas bottom-heavy particles align along the flow direction and move upwards against the flow [5].

2 Gravity

2.1 Sedimentation of Active Particles

In this section the sedimentation profile of spherical active Janus colloids in a H$_2$O$_2$ solution is first experimentally obtained and then theoretically described. A schematic sketch of the experimental setup is given in Fig. 1a. However, before discussing the behaviour of active particles in gravitational field it is reasonable to take a brief look at passive particles. At thermal equilibrium a dilute population of colloids with radius $R$ and mass $m$ will exhibit a steady Boltzmann distribution profile

$$\Omega(z) = \Omega_0 \exp\left(-\frac{z}{\delta_0}\right)$$

with $\delta_0 = k_BT/mg$, where $\delta_0$ is the sedimentation length, $T$ the bath temperature and $\Omega_0$ the density on the ground. The translational diffusion coefficient and the sedimentation length are connected via $D = \delta_0 mg/\gamma$ with the translational Stokes
friction \( \gamma \). Now the Janus active colloids exhibit nearly the same behaviour when reaching a stationary state. The density profile is shown in Fig. 1b.

The density profile of the active particles is very well fitted by an exponential decay

\[
\Omega(z) = \Omega_0 \exp(-z/\delta_{\text{eff}}),
\]

where \( \delta_{\text{eff}} \) is the effective sedimentation length. As for passive particles we find the connection \( D_{\text{eff}} = \delta_{\text{eff}} m g / \gamma \). To compare active motion to thermal translational diffusion we introduce the Pécelt number which is defined as the ratio between the advective transport rate and the diffusive transport rate. This gives the Pécelt number

\[
Pé = \frac{v_0 R}{D},
\]

i.e. for high Pécelt numbers activity dominates. From [6] it can be found that the effective diffusion coefficient is

\[
D_{\text{eff}} = D (1 + 2 Pe^2 / 9).
\]

Thus the effective sedimentation length depends strongly on the activity of the particles. To understand this phenomenon it is necessary to take a look at theory. For instance, the Fokker–Planck–equation for the probability distribution function \( \rho(r, \mathbf{e}, t) \), which includes the intrinsic colloidal orientation \( \mathbf{e} \), has to be introduced

\[
\frac{\partial}{\partial t} \rho(r, \mathbf{e}, t) + \nabla J_{\text{trans}} + \mathfrak{R} J_{\text{rot}} = 0,
\]

with the respective translational and rotational current

\[
J_{\text{trans}} = -D \nabla \rho + (v_0 \mathbf{e} + m g / \gamma) \rho, \quad J_{\text{rot}} = -D_r \mathfrak{R} \rho,
\]

and the rotational operator \( \mathfrak{R} = \mathbf{e} \times \nabla e \). The timescale \( \tau_r = D_r^{-1} \) with the rotational diffusion coefficient \( D_r = k_B T / (8 \pi \eta R^3) \), where \( \eta \) is the fluid’s viscosity, describes the decay of a fixed orientation \( \mathbf{e} \) at time \( t = 0 \) s due to the rotational diffusion. The contributions of the translational current \( J_{\text{trans}} \) result from the thermal diffusion, the activity and the gravity. The
rotational current \( J^\text{rot} \) is purely diffusive. To solve the Fokker–Planck–equation (Eq. 5) a multipole expansion ansatz is used

\[
\rho(r, e, t) = \frac{1}{4\pi} \left( \Omega(r, t) + 3P(r, t) + \frac{15}{2} \mathbf{Q}(r, t) \cdot (e \otimes e - \frac{1}{3} I) + \ldots \right).
\]

The total probability density \( \Omega(r, t) = \int \rho(r, e, t) d^2e \) and the respective dipole density moment and quadrupole density moment \( \mathbf{P}(r, t) = \int e \rho(r, e, t) d^2e \) and \( \mathbf{Q}(r, t) = \int (e \otimes e - \frac{1}{3} I) \rho(r, e, t) d^2e \) are introduced. By multiplying the Fokker–Planck–equation (5) with \( e, \mathbf{e}, \text{etc.} \) and integrating over all orientations \( e \) one obtains differential equations for the total probability density \( \Omega(r, t) \), the dipole density moment \( \mathbf{P}(r, t) \), etc. It is not possible to obtain a closed hierarchy of the equations since the leading moment couples always to a higher moment through the active drift term. Here, the dynamic equations for the total density \( \Omega(r, t) \) and dipole density \( \mathbf{P}(r, t) \) will be discussed

\[
\begin{align*}
\partial_t \Omega + \nabla \cdot (-D\nabla + mg/\gamma)\Omega + v_0 \nabla \mathbf{P} &= 0, \\
\partial_t \mathbf{P} + \nabla \cdot (-D\nabla + mg/\gamma)\mathbf{P} + 2D_\gamma \mathbf{P} + v_0 \nabla \cdot \mathbf{Q} + \frac{v_0}{3} \nabla \Omega &= 0.
\end{align*}
\]

In Eq. (10) the time derivation \( \partial_t \mathbf{P} \) can be neglected for time scales larger than \( t_\gamma = D_\gamma^{-1} \). Further, we assume small contributions from \( \mathbf{Q} \) and neglect it. And finally the moments \( \rho \) and \( \mathbf{P} \) vary smoothly in space, so the space derivation can be neglected. With these assumptions Eq. (10) is reduced to

\[
\mathbf{P}(r, t) \approx -\frac{v_0}{6D_\gamma} \nabla \Omega.
\]

Plugging this expression in Eq. (9) we obtain

\[
\left(-D_{\text{eff}} \nabla^2 + \frac{mg}{\gamma} \right) \Omega = 0
\]

which can be solved exactly by Eq. (2). Not only we obtained from a basic mathematical approach the sedimentation profile for active particles we can now explain why the sedimentation length increases with the activity by considering the Polarization \( \mathbf{P}_z = \langle \cos \theta \rangle \Omega \) with the angle \( \theta = \mathbf{e}_z \cdot \mathbf{e} \). The mean orientation arrives at

\[
\langle \cos \theta \rangle \approx \frac{2a}{9} \frac{P_e}{\delta_0 (1 + 2Pe^2/9)}
\]

which is not zero – for active particles, i.e. non-vanishing Peclet number \( Pe \) – as it is the case for passive particles, but the colloids develop a polar order along the vertical. Thus with higher activity the orientation \( e \) of more colloids align to the vertical axis and they bias to swim upwards.

### 2.2 Gravitaxis caused by Shape Asymmetry

In general “taxis” describes the alignment of a properly defined director axis along an external field, i.e. in this case gravitaxis means the response of the microswimmer to the gravitational field. In this part we consider a L-shaped artificial microswimmer which has a homogenous mass distribution and is asymmetric shaped, see Fig. 2a. The particles are covered with a thin Au coating on the front side of the short arm. Self-diffusiophoretic motion can be induced with laser illumination with the intensity \( I \) due to the local heating of the coated side. When the particles are suspended in a binary mixture of water and 2,6-lutidine at critical composition, this heating results in a phoretic propulsion in the direction normal to the coated metal cap which depends on the intensity. The motion of the particle is restricted to two dimensions by using a sample height of \( h = 7 \mu m \). The gravitational force can be varied by mounting the sample cell with an angle \( \alpha \) relatif to the horizontal plane – in the following the inclination angle is determined to \( \alpha = 10.67^\circ \).
2.2 Gravitaxis caused by Shape Asymmetry

![Geometrical sketch of an ideal L-shaped particle coated with Au indicated with the orange line](a)

**Figure 2:** (a) Geometrical sketch of an ideal L-shaped particle coated with Au indicated with the orange line. The dimensions of the particle are $a = 9 \mu m$, $b = 3 \mu m$ and the coordinates $x_{CM} = -2.25 \mu m$, $y_{CM} = 3.75 \mu m$ of the center of mass (with the origin of coordinates in the bottom right corner of the particle and when the Au coating is neglected). $\hat{u}_\parallel$ and $\hat{u}_\perp$ are particle fixed unit vectors. (b) Experimental setup with an inclination angle $\alpha = 10.67^\circ$.

The motion of an active particle was measured for various intensities $I$, i.e. for the different self-propulsion strengths $P^*$. This is shown in Fig. 3. The passive particle swims straight downward with a fixed orientation angle $\phi = -34^\circ$ and the shorter arm at the bottom (trajectory 1). For sufficiently high self-propulsions the particle is able to overcome the sedimentation and it performs a straight upward swimming with again a stable orientation $\phi$ (trajectory 2-4). With further increasing self-propulsion the orientation $\phi$ exceeds $90^\circ$ and thus the particle swims straight downward (trajectory 5). The orientation $\phi$ shows a monotonic dependence on the self-propulsion strength $P^*$ till a critical self-propulsion is reached. Above this critical self-propulsion the particle performs a trochoid-like motion (trajectory 6).

To obtain a theoretical understanding of the behaviour of the asymmetric shaped active particles we study the Langevin equations

\[
\dot{r} = \left( \frac{P^*}{b} \right) (D_T \hat{u}_\perp + lD_C) + \beta D_T F_G + \zeta_r, \\
\dot{\phi} = \left( \frac{P^*}{b} \right) (lD_R + D_C \cdot \hat{u}_\perp) + \beta D_C \cdot F_G + \zeta_\phi,
\]

for the time-dependent center of mass position $r(t) = (x(t), y(t))$ and orientation angle $\phi(t)$. Here, the gravitational force

![Experimental trajectories for the same inclination angle $\alpha$ and increasing illumination intensity $I$.](c)

**Figure 3:** Experimental trajectories for the same inclination angle $\alpha$ and increasing illumination intensity $I$. 
Gravitaxis caused by Shape Asymmetry

$\mathbf{F}_G$, the translational short-time diffusion tensor $\mathbf{D}_T$, the translational-rotational coupling vector $\mathbf{D}_C$, the inverse effective thermal energy $\beta = 1/(k_B T)$ and the Brownian noise terms $\zeta$, and $\zeta_{\phi}$ are involved. The dynamics of the asymmetric particle is determined by a self-propulsion term which is proportional to the self-propulsion strength $P^*$, a gravitational force term and a Brownian noise term. Up to a threshold value $P^*$ the effective torque originating from the self-propulsion can be compensated by the gravitational torque so that no net rotation takes place and the trajectories are straight. The orientation angle $\phi$ converges to a fixed angle

$$
\phi_\infty = \phi(t \to \infty) = \phi_\infty = -\arctan \left( \frac{D_C \perp}{D_C \parallel} \right) + \arcsin \left( \frac{P^*}{\beta b F_G} \frac{D_C \perp + |D_R|}{\sqrt{D_C \parallel^2 + D_C \perp^2}} \right).
$$

(16)

The converged orientation $\phi_\infty$ consists of a superposition of the passive case (left term in Eq. (16)) and a correction due to self-propulsion (right term in Eq. (16)). In Fig. 4 the long-time orientation $\phi_\infty$ is plotted over the self-propulsion strength $P^*$ and is compared with the measured values in the regime of straight motion. A very good agreement between theory and experiment is obtained.

Beyond the regime of straight motion a trochoid-like motion occurs. The effective torque originating from the self-propulsion can no longer be compensated by the restoring torque generated from the gravitational field. The different types of motion depend on the self-propulsion $P^*$ but also on the inclination angle $\alpha$. To determine each regime the Langevin equations (14,15) are solved for various $P^*$ and $\alpha$. The state diagram is shown in Fig. 5a. These calculations reveal three types of motions. First, the straight downward swimming (SDS) which can be divided into a straight downward swimming in negative x-direction (SDS-) for small self-propulsion strengths $P^*$ and a straight downward swimming in positive x-direction (SDS+). Second, further increasing of the self-propulsion $P^*$ leads to negative gravitaxis, i.e. straight upwards swimming (SUS). And Third, for self-propulsions $P^*$ higher than $P^*_{\text{crit}}$ the particle exhibits a trochoid-like motion (TLM). Indeed, the experimental observed types of motion correspond to those in the theoretical obtained state diagram (see Box in Fig. 5a). Analysing Eq. (14), the state diagram should depend strongly on the length of the effective lever arm. Therefore the Au is coated in a way that it slightly extends over the front face of the L-particles to one of the lateral planes. This results in a shift of the effective self-propulsion force and changes the lever arm to $l = -1.65 \, \mu\text{m}$. The state diagram for the L-particle with the varied lever arm is shown in Fig. 5b. Interestingly, no evidence for negative gravitaxis, i.e. straight upward swimming, could be find in neither the theoretical prediction nor the experimental measurements. This suggests that negative gravitaxis depends not only on the strength of the self-propulsion but also on the position where the effective force acts on the body.

Figure 4: Long-time orientation $\phi_\infty$ in the regime of straight motion as a function of the self-propulsion strength $P^*$. The red squares and green bullets correspond to straight upward and downward swimming, respectively.
3 External Flow

3.1 Symmetric Swimmer in Poiseuille Flow

We consider a spherical microswimmer with swimming direction $\mathbf{e}$ and velocity $v_0$ moving in Poiseuille flow through a microchannel. Poiseuille flow is characterized by a laminar flow through a pipe of constant circular cross-section in an incompressible and Newtonian fluid with no acceleration of the fluid in the pipe. To neglect thermal translational diffusion we assume a large Péclet number $\text{Pe} \gg 1$, i.e. activity of the particle dominates. Furthermore, we neglect rotational diffusion by assuming a large persistence number $\text{Pe}_r = v_0 / (D_r R) \gg 1$, so the particle moves in a persistent motion in one direction. The flow strength $v_f$ shall be sufficiently large so that hydrodynamical interactions with the bounding walls can be neglected. The microchannel is cylindrical shaped, so the pressure-driven Poiseuille flow along the $z$-direction amounts to

$$v_P = v_f \left[ 1 - \left( \frac{x}{R_{ch}} \right)^2 \right] \mathbf{e}_z,$$

where $R_{ch}$ is the channel radius. We concentrate here on the microswimmer swimming in the $xz$-plane, the setup is shown in Fig. 6.

The swimming direction of the active microswimmer is in contrast to a passive one not only advected but it is also able to

Figure 5: (a) State diagram of the motion of an active L-shaped particle with lever arm $l = -0.75 \, \mu m$ under gravity. The types of motion are straight downward swimming (SDS), straight upward swimming (SUS), and trochoid-like motion (TLM). (SDS+/−) indicates a drift in positive/negative $x$-direction. (b) State diagram of the motion of an active L-shaped particle with lever arm $l = -1.65 \, \mu m$ under gravity. Notice that here the regime of straight upward swimming is missing.

Figure 6: A microswimmer moves in the $xz$-plane through a microchannel with the channel radius $R_{ch}$ under Poiseuille flow $v_P$ which is directed in the $z$-axis. The local vorticity $\Omega_P$ results due to the decreasing strength for higher values of $x$. The vorticities rotate the swimmer orientation $\mathbf{e}$ which is quantified by the angle $\psi$. 

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cross streamlines. The dynamics of the position vector \( r(t) = (x(t), z(t)) \) and the orientation vector \( e(t) \) follow

\[
\frac{dr}{dt} = v_0 e + v_P(r),
\]

\[
\frac{de}{dt} = \frac{1}{2} \Omega_P \times e.
\]

The dynamics of the position \( r(t) \) consists of a superposition of the active motion directed to \( e \) and the Poiseuille flow \( v_P \). Whereas the dynamics of the orientation is due to the spherical symmetry of the particle only determined by the rotation of the vorticity vector \( \Omega_P \). The equations of motion become for the position \( x(t) \) and the angle \( \psi \)

\[
\dot{x} = -v_0 \sin \psi,
\]

\[
\dot{\psi} = \frac{v_f}{R_{ch}^2} x.
\]

\[
\Rightarrow 0 = \dot{\psi} + \frac{v_f v_0}{R_{ch}^2} \psi.
\]

We see the orientation angle \( \psi \) obeys the same equation as the mathematical pendulum. So the particle moves on a swinging trajectory while swimming upstream. This becomes clear by considering Fig. 6. If the particle is in the upper half of the channel the orientation \( e \) rotates counterclockwise towards the centerline. When the particle crosses the centerline the rotation inverts and the orientation rotates clockwise, i.e. again towards the centerline. We can formally introduce a Hamiltonian as a constant of motion for the coordinates \( x \) and \( \Psi \)

\[
H = \frac{v_f}{2R_{ch}^2} x^2 + v_0 (1 - \cos \Psi)
\]

and derive the equation of motions

\[
\dot{x} = \frac{\partial H}{\partial \Psi} \quad \text{and} \quad \dot{\Psi} = -\frac{\partial H}{\partial x}.
\]

Two types of motion occur for different strengths of \( H \). First, swinging solutions of the microswimmer which correspond to \( H < 2v_0 \) and second, the tumbling motion which corresponds to \( H > 2v_0 \). However, to determine the full trajectory of the swimmer one has to solve also the dynamic equation for the \( z \) component

\[
\dot{z} = v_f \left[ 1 - \left( \frac{x}{R_{ch}} \right)^2 \right] - v_0 \cos \Psi.
\]

In Fig. 7 several swimming trajectories for different flow strengths \( \tau = \frac{v_f}{v_0} \) are presented.

The swimmer always moves upstream when \( v_f < v_0 - H \) shown in Fig. 7a. The sinusoidal motion is observable even if the

![Figure 7: Swimming trajectories for different flow strengths \( \tau = \frac{v_f}{v_0} \). The orientation of the microswimmer is indicated by arrows, it starts always at \( z = 0 \). The blue and green trajectories differ in the choice of initial conditions. Note, the direction of the \( z \)-axis changes in (e).](image-url)
particle crashes into the wall (green trajectory). For flow strengths $v_f > v_0 + 2H$ the swimmer moves always downstream. He shows both a tumble motion close to the wall (green trajectory in Fig. 7c) and a swinging motion (blue trajectory). In between $v_0 - H < v_f < v_0 + 2H$, both an effective up- and downstream motion exists but the swimmer is always oriented upstream, shown in Fig. 7b. This trajectory deviates strongly from a sinusoidal oscillation. In summary, although the swimmer is spherically symmetric a upstream motion occurs. The swimmer is able to align his orientation along the gradient of the Poiseuille flow.

3.2 Rheotaxis caused by Inhomogenous Mass Distribution

We consider a bottom-heavy particle with radius $R$, i.e. the center of mass and the geometrical center differ, in a capillary connected to a syringe pump imposing flow in the $x$-direction. The particles are made of a polymer with a hematite cube protruding out. The particle is dispersed in a water-based solution containing hydrogen peroxide and tetramethylammonium hydroxide to adjust the pH to $\sim 8.5$. The particle and the schematic experimental setup are shown in Fig. 8.

Under bright-field illumination the particle is at equilibrium with solvent and rests at a gravitational height $h_g = \delta_0 = k_B T / (m g) \approx 100$ nm. When illuminating with blue light the particle starts to self-propel. In the case of a passive particle (no blue illumination) the shear flow created in the vicinity of the lower solid surface exerts a torque on the particle and thus beside the translational motion it starts to rotate. So the particle translates and rotates on average at an altitude $y_g = R + h_g$.

It can be shown that the translation and rotation at the altitude $y_g$ are completely decoupled by freezing the rotation of the particle with a uniform magnetic field. No modification of the translational velocity can be observed. By illuminating with blue laser the particle becomes active and it stops rolling. It performs a turn so that the protrusion faces the imposed flow. Then it migrates upstream once activated. The positive rheotaxis has to do with how the active particle aligns to the flow. For simplicity we assume that the hematite cube is a fixed pivot point. The flow exerts a Stokes drag $F_s = 6\pi \eta R v$ on the particle leading to a torque $M_s = 6\pi \eta R^2 v \sin \theta e_z$ with the angle $\theta$ defined as in Fig. 9.

The problem is due to the low Reynolds-number $Re \sim 10^{-5}$ formally identical to an overdamped Brownian pendulum in an effective field with the effective potential

$$U_{eff} = -6\pi \eta R^2 v \cos \theta.$$  \hspace{1cm} (26)

The angular distribution is then given by the Fokker-Planck-equation

$$\partial_t P(\theta, t) = D_\theta \partial_\theta \left[ \partial_\theta P(\theta, t) + \beta 6\pi \eta R^2 v \sin \theta P(\theta, t) \right],$$  \hspace{1cm} (27)

with the inverse effective thermal energy $\beta = 1/(k_B T)$. The time evolution of the angular probability distribution consists

Figure 8: (a) A syringe pump is connected to the capillary and induces a flow along the $x$-direction. The particle experiences a shear flow $v = \gamma y$ where $\gamma$ is the local shear rate. (b) Spherical polymer with a hematite cube protruding out (right).
4 Conclusion

In the first part we investigated the behaviour of active particles under gravity. Spherical active particles show equally to passive particles an exponential density profile but with an effective sedimentation length which depends strongly on the activity of the particles. These particles develop a polar order aligning their orientation $\mathbf{e}$ along the vertical. Nevertheless, are not able to show negative gravitaxis. This occurs by breaking the symmetry of the particles. If the effective torque originating from the shape asymmetry and the restoring torque generated by the gravity can compensate each other a steady orientation is reached and the particle swims upwards.

In the second part the dynamics of active particles in external flow was investigated. From a theoretical point of view we predict that spherical symmetric particles under Poiseuille flow in a microchannel can exhibit a swinging upstream motion for slow flow strength and a tumbling motion downstream for strong flows. The rheotactic behaviour of the bottom-heavy particle results due to his alignment of the tail-head direction to the imposed flow.

Quellen


