Problem Sheet 10
Solid State Theory
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Fakultät für Physik, Universität Stuttgart
Prof. Dr. R. Hilfer

Problem 1) **(4 Points)**

Show that a parallel-plate capacitor made up of two parallel layers of material - one layer with dielectric constant ε, zero conductivity, and thickness d, and the other layer with zero dielectric constant, finite conductivity σ, and thickness qd - behaves as if the space between the condenser plates were filled with a homogeneous dielectric with dielectric constant

$$\varepsilon_{\text{eff}} = \frac{\varepsilon(1+q)}{1-(\mathrm{i}\varepsilon\omega q)/(4\pi\sigma)},$$

where ω is the angular frequency. Values of ε_{eff} as high as 10^4 or 10^5 caused largely by this Maxwell-Wagner mechanism are sometimes found, but the high values are always accompanied by large ac losses.

Problem 2) **(4 Points)**

The wave function of the hydrogen atom in its ground state (1s) reads $\psi = \mathrm{e}^{-r/a_0}/\sqrt{\pi a_0^3}$, where $a_0 = \hbar^2/(me^2) = 0.529 \times 10^{-8}\,\text{cm}$. The charge density is $\rho(x,y,z) = -e|\psi|^2$, according to the statistical interpretation of the wave function. Show that for this state $\langle r^2 \rangle = 3a_0^2$, and calculate the molar diamagnetic susceptibility of atomic hydrogen (which is given by $2.36 \times 10^{-6}\,\text{cm}^3\,\text{mol}^{-1}$).

Problem 3) **(4 Points)**

The spin susceptibility of a conduction electron gas at absolute zero may be discussed by another method. Let

$$N^+ = \tfrac{1}{2}N(1+\zeta), \qquad N^- = \tfrac{1}{2}N(1-\zeta)$$

be the concentrations of spin-up and spin-down electrons.

(a) Show that in a magnetic field B the total energy of the spin-up band in a free electron gas is

$$E^+ = E_0(1+\zeta)^{5/3} - \tfrac{1}{2}N\mu B(1+\zeta),$$

where $E_0 = \frac{3}{10}N\varepsilon_F$, in terms of the Fermi energy ε_F in zero magnetic field. Find a similar expression for E^-.

(b) Minimize $E_{\text{total}} = E^+ + E^-$ with respect to ζ and solve for the equilibrium value of ζ in the approximation $\zeta \ll 1$. Go on to show that the magnetization is given by $M = 3N\mu^2 B/(2\varepsilon_F)$, in agreement with Equation (42) in Chapter 10 of the Lecture.