Problem Sheet 9
Solid State Theory
Summer Semester 2021

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Problem 1) **(4 Points)**

On the two-fluid model of a superconducter we assume that at temperatures $0 < T < T_c$ the current density may be written as the sum of the contributions of normal and superconducting electrons: $\vec{j} = \vec{j}_N + \vec{j}_S$, where $\vec{j}_N = \sigma_0 \vec{E}$ and $\vec{j}_S$ is given by the London equation. Here σ_0 is an ordinary normal conductivity, decreased by the reduction in the number of normal electrons at temperature T as compared to the normal state. Neglect inertial effects on both $\vec{j}_N$ and $\vec{j}_S$.

(a) Show from the Maxwell equations that the dispersion relation connecting wavevector $\vec{k}$ and frequency ω for electromagnetic waves in the superconductor is

$$k^2c^2 = 4\pi\sigma_0\omega\mathrm{i} - c^2\lambda_L^{-2} + \omega^2; \qquad \text{(CGS)}$$

or

$$k^2c^2 = (\sigma_0/\epsilon_0)\omega\mathrm{i} - c^2\lambda_L^{-2} + \omega^2, \qquad \text{(SI)}$$

where λ_L^2 is given by Equation (14a), in Chapter 8 of the Lecture, with n replaced by n_S. Recall that $\mathrm{curl}\,\mathrm{curl}\,\vec{B} = -\nabla^2\vec{B}$.

(b) If τ is the relaxation time of the normal electrons and n_N is their concentration, show by use of the expression $\sigma_0 = n_N e^2\tau/m$ that at frequencies $\omega \ll 1/\tau$ the dispersion relation does not involve the normal electrons in an important way, so that the motion of the electrons is described by the London equation alone. The supercurrent short-circuits the normal electrons. The London equation itself only holds true if $\hbar\omega$ is small in comparison with the energy gap. *Note:* The frequencies of interest are such that $\omega \ll \omega_p$, where ω_p is the plasma frequency.

Problem 2) **(4 Points)**

Consider a semiclassical model of the ground state of the hydrogen atom in an electric field normal to the plane of the orbit (Figure 1), and show that for this model $\alpha = a_H^3$, where a_H is the radius of the unperturbed orbit.
Note: If the applied field is in the x direction, then the x component of the field of the nucleus at the displaced position of the electron must be equal to the applied field. The correct quantum-mechanical result is larger than this by the factor $\frac{9}{2}$. (We are speaking of α_0 in the expansion $\alpha = \alpha_0 + \alpha_1\vec{E} + \cdots$) We assume $x \ll a_H$. One can also calculate α_1 on this model.

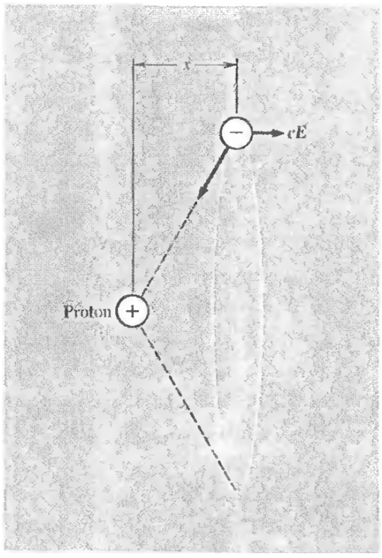

Figure 1: An electron in a circular orbit of radius a_H is displaced a distance x on application of an electric field $\vec{E}$ in the $-x$ direction. The force on the electron due to the nucleus is e^2/a_H^2 in CGS or $e^2/4\pi\epsilon a_H^2$ in SI. The problem assumes $x \ll a_H$.

Problem 3) **(4 Points)**

Show that the polarizability of a conducting metallic sphere of radius a is $\alpha = a^3$. This result is most easily obtained by noting that $\vec{E} = 0$ inside the sphere and then using the depolarization factor $4\pi/3$ for a sphere (Figure 2). The result gives values of α of the order of magnitude of the observed polarizabilities of atoms. A lattice of N conducting sphere per unit volume has dielectric constant $\epsilon = 1 + 4\pi N a^3$, for $Na^3 \ll 1$. The suggested proportionality of α to the cube of the ionic radius is satisfied quite well for alkali and halogen ions. To do the problem in SI, use $\frac{1}{3}$ as the depolarization factor.

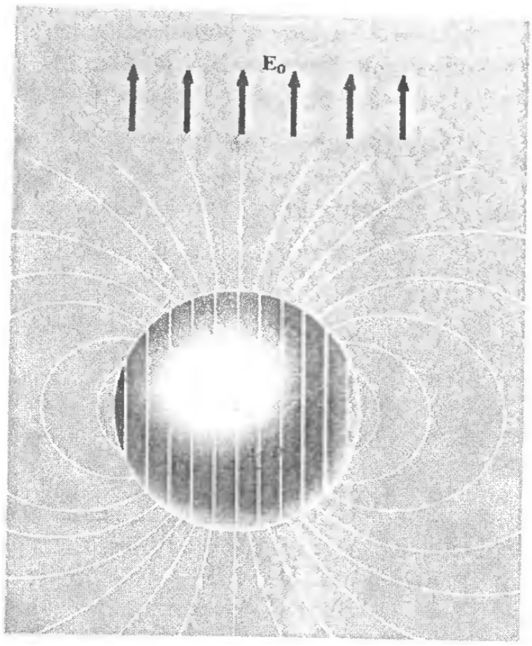

Figure 2: The total field inside a conducting sphere is zero. If a field $\vec{E}_0$ is applied externally, then the field $\vec{E}_1$ due to surface charges on the sphere must just cancel $\vec{E}_0$, so that $\vec{E}_0 + \vec{E}_1 = 0$ within the sphere. But $\vec{E}_1$ can be simulated by the depolarization field $-4\pi\vec{P}/3$ of a uniformly polarized sphere of polarization $\vec{P}$. Relate $\vec{P}$ to $\vec{E}_0$ and calculate the dipole moment $\vec{p}$ of the sphere. In SI the depolarization field is $-\vec{P}/3\epsilon_0$.