Problem Sheet 8
Solid State Theory
Summer Semester 2021

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Problem 1) **(4 Points)**

The penetration equation reads $\lambda^2 \nabla^2 B = B$, where λ is the penetration depth.

(a) Show that $B(x)$ inside a superconducting plate perpendicular to the x axis with center at $x = 0$ and of thickness δ is given by

$$B(x) = B_a \cdot \frac{\cosh(x/\lambda)}{\cosh(\delta/2\lambda)},$$

where B_a is the field outside the plate and parallel to it.

(b) The effective magnetization $M(x)$ in the plate is defined by $B(x) - B_a = 4\pi M(x)$. Show that, in CGS,

$$4\pi M(x) = -(\delta^2 - 4x^2)B_a/(8\lambda^2) \qquad \text{for } \delta \ll \lambda.$$

In SI we replace the 4π by μ_0.

Problem 2) **(4 Points)**

(a) Using the result of Problem 1(b), show that the free energy density at $T = 0\,\mathrm{K}$ within a superconducting film of thickness δ in an external magnetic field B_a is

$$F_S(x, B_a) = U_S(0) + (\delta^2 - 4x^2)B_a^2/(64\pi\lambda^2) \qquad \text{for } \delta \ll \lambda.$$

In SI the factor π is replaced by $\frac{1}{4}\mu_0$. We neglect a kinetic energy distribution.

(b) Show that the magnetic contribution to F_S when averaged over the thickness of the film is $B_a^2(\delta/\lambda)^2/(96\pi)$.

(c) Show that the critical field of the thin film is proportional to $(\lambda/\delta)H_c$, where H_c is the bulk critical field, if we consider only the magnetic contribution to U_S.

Problem 3) **(4 Points)**

(a) Take the time derivative of the London equation (10) from Chapter 8 of the Lecture to show that $\partial\vec{j}/\partial t = (c^2/(4\pi\lambda_L^2))\vec{E}$.

(b) If $m\mathrm{d}\vec{v}/\mathrm{d}t = q\vec{E}$, as for free carriers of charge q and mass m, show that $\lambda_L^2 = mc^2/(4\pi n q^2)$.