Problem Sheet 7
Solid State Theory
Summer Semester 2021

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Problem 1) **(4 Points)**

The Kramers-Kronig relations are consistent with the principle that an effect not precede its cause. Consider a delta-function force applied at time $t = 0$:

$$F(t) = \delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t}\, \mathrm{d}\omega, \tag{1}$$

whence $F_\omega = 1/2\pi$.

(a) Show by direct integration or by use of the KK relations that the oscillator response function

$$\alpha(\omega) = (\omega_0^2 - \omega^2 - \mathrm{i}\omega\rho)^{-1} \tag{2}$$

gives zero displacement, $x(t) = 0$, for $t < 0$ under the above force. For $t < 0$ the contour integral may be completed by semicircle in the upper half-plane.

(b) Evaluate $x(t)$ for $t > 0$. Note that $\alpha(\omega)$ has poles at $\pm(\omega_0^2 - \frac{1}{4}\rho^2)^{1/2} - \frac{1}{2}\mathrm{i}\rho$, both in the lower half-plane.

Problem 2) **(4 Points)**

By comparison of $\alpha'(\omega)$ in the limit $\omega \to \infty$, where $\alpha'(\omega)$ is given by Eqs. (9) and (11a) from Chapter 7 of the Lecture, show that the following sum rule for the oscillator strengths must hold:

$$\textstyle\sum_j f_j = \frac{2}{\pi} \int_0^{\infty} s\alpha''(s)\, \mathrm{d}s. \tag{3}$$

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Problem 3) **(4 Points)**

Consider an electromagnetic wave in vacuum, with field components of the form

$$E_y(\mathrm{inc}) = B_z(\mathrm{inc}) = A\mathrm{e}^{\mathrm{i}(kz-\omega t)}. \tag{4}$$

Let the wave be incident upon a medium of dielectric constant ϵ and permeability $\mu = 1$ that fills the half-space $x > 0$. Show that the reflectivity coefficient $r(\omega)$ as defined by $E(\mathrm{refl}) = r(\omega)E(\mathrm{inc})$ is given by

$$r(\omega) = \frac{n + \mathrm{i}K - 1}{n + \mathrm{i}K + 1}, \tag{5}$$

where $n + \mathrm{i}K = \epsilon^{1/2}$, with n and K real. Show further that the reflectance is

$$R(\omega) = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}. \tag{6}$$