Problem Sheet 6
Solid State Theory
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Fakultät für Physik, Universität Stuttgart
Prof. Dr. R. Hilfer

Problem 1) **(4 Points)**

Consider a semi-infinite plasma on the positive side of the plane $z = 0$. A solution of Laplace's equation $\nabla^2\phi = 0$ in the plasma is $\phi_{\mathrm{i}}(x, z) = A\cos(kx)\mathrm{e}^{-kz}$, whence $E_{z\mathrm{i}} = kA\cos(kx)\mathrm{e}^{-kz}$ and $E_{x\mathrm{i}} = kA\sin(kx)\mathrm{e}^{-kz}$.

(a) Show that in the vacuum $\phi_{\mathrm{o}}(x, z) = A\cos(kx)\mathrm{e}^{kz}$ for $z < 0$ satisfies the boundary condition that the tangential component of $\vec{E}$ be continuous at the boundary; that is, find $E_{x\mathrm{o}}$.

(b) Note that $\vec{D}_{\mathrm{i}} = \epsilon(\omega)\vec{E}_{\mathrm{i}}$ and $\vec{D}_{\mathrm{o}} = \vec{E}_{\mathrm{o}}$. Show that the boundary condition that the normal component of $\vec{D}$ be continuous at the boundary requires that $\epsilon(\omega) = -1$, whence from Eq. (10) in Chapter 6 of the Lecture we have the Stern-Ferrell result:

$$\omega_s^2 = \tfrac{1}{2}\omega_p^2 \tag{1}$$

for the frequency ω_s of a surface plasma oscillation.

Problem 2) **(4 Points)**

The frequency of the uniform plasmon mode of a sphere, is determined by the depolarization field $\vec{E} = -4\pi\vec{P}/3$ of a sphere, where the polarization is $\vec{P} = -ne\vec{r}$, with $\vec{r}$ as the average displacement of the electrons of concentration n. Show from $\vec{F} = m\vec{a}$ that the resonance frequency of the electron gas is $\omega_0^2 = 4\pi ne^2/(3m)$. Because all electrons participate in the oscillation, such an excitation is called a collective excitation or collective mode of the electron gas.

Problem 3) **(4 Points)**

(a) Find what Eq. (56) in Chapter 6 of the Lecture becomes when $\epsilon(\infty)$ is taken into account.

(b) Show that there is a solution of Eq. (55) in Chapter 6 which at low wavevector is $\omega = cK/\sqrt{\epsilon(0)}$, which is what you expect for a photon in a crystal of refractive index $n^2 = \epsilon$.