Problem Sheet 3
Solid State Theory
Summer Semester 2021

Fakultät für Physik, Universität Stuttgart

Prof. Dr. R. Hilfer

Problem 1) **(3 Points)**

The relation between energy and wave vector of free electrons is $E(k) = \hbar^2 k^2/(2m)$ where $k = |\vec{k}|$. Compute the density of states functions $D(k)$ and $D(E)$ for the three dimensional free electron gas from their definitions $D(\vec{k})\,\mathrm{d}^3\vec{k} = D(k)\,\mathrm{d}k = D(E)\,\mathrm{d}E$.

Problem 2) **(6 Points)**

Recall the free electron model of a metal. Recall the definitions of Fermi energy and Fermi temperature.

(a) Why do metals held at room temperature feel cold to touch although their Fermi temperatures are much higher than room temperature?

(b) A d-dimensional sample of volume L^d contains N free electrons. Show that the Fermi energy is given by

$$E_F = \frac{\hbar^2}{2mL^2}(Na_d)^{2/d}.$$

Give the numerical values of a_d for $d = 1, 2, 3$.

(c) Show that the density of states at the Fermi energy is

$$D(E_F) = \frac{Nd}{2L^d E_F}.$$

(d) Assuming applicability of the free electron model to a one-dimensional organic conductor with unit cells of 0.8 nm estimate the Fermi energy and Fermi temperature if each unit cell contributes one mobile electron.

(e) Consider relativistic electrons where $E = c|\vec{p}|$. Calculate the Fermi energy as a function of density for electrons in $d = 1, 2, 3$ and calculate the density of states at the Fermi energy in each case.

Please turn page!

Problem 3) **(3 Points)**

(a) Show that for a simple square lattice ($d = 2$) the kinetic energy of a free electron at a corner of the first zone is higher by a factor 2 than that of an electron at midpoint of a side face of the first zone.

(b) What is the corresponding factor for a simple cubic lattice?

(c) What might the result in (b) imply for divalent elements?